

## AENEID I AND .618?

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SINCE PAUL MAURY in 1944 propounded his theory that Virgil made use of Neo-Pythagorean symbolic numbers in the composition of the *Eclogues*,<sup>1</sup> and Guy Le Grelle, s.j., extending Maury's work, suggested that the first book of the *Georgics* was constructed in accordance with the Divine Proportion, .618,<sup>2</sup> the mathematical approach to structural analysis has become a major aspect of Virgilian scholarship. Although the results produced by mathematical analysis are interesting, the method itself has not won unqualified approval. This is especially the case with the mathematical sections of Professor G. E. Duckworth's *Structural Patterns and Proportions in Vergil's Aeneid: A Study in Mathematical Composition*, published in 1962. It is his thesis that Virgil deliberately and consistently made use of the arithmetical equivalent of Euclid 2.11 and 6.30, the Golden Mean ratio or Golden Section, .618, as the basis for organizing his narrative: "he wrote the short passages with mathematical symmetry and combined them into increasingly larger units, with every part of every book containing exact or approximate Golden Mean ratios ranging from .60 to .636" (p. 73). Professor Duckworth has found at least 1044 Golden Mean ratios in the *Aeneid*.

The supporters of Professor Duckworth's theory have been enthusiastic.<sup>3</sup> R. J. Getty went so far as to say "He has brought to the analysis of the *Aeneid* a principle of much aesthetic importance and has defended it stoutly. What is now at stake is not his reputation, but that of his critics if they express their disbelief without being able to refute him." Yet much can be and has been said by those who find themselves unconvinced.<sup>4</sup> Most reviewers point out that the validity of the mathematical thesis rests upon the logical relation between suggested mathematical units and coherently interrelated passages of narrative. As yet, however, no systematic attempt has been made to examine in detail the subject-divisions which Professor Duckworth uses. This I propose to do, using the subject-divisions and ratios suggested for *Aeneid* 1. As a preface to the study, I shall comment briefly on the Golden Mean ratio

<sup>1</sup>"Le secret de Virgile et l'architecture des Bucoliques," *BAGB* (1944) 71-147.

<sup>2</sup>"Le premier livre des *Géorgiques*, poème pythagoricien," *LEC* 17 (1949) 139-235.

<sup>3</sup>See, e.g., R. J. Getty, *Vergilius* 9 (1963) 20; R. B. Lloyd, *AJP* 85 (1964) 71-77; and E. C. Woodcock, *PACA* 6 (1963) 50-51.

<sup>4</sup>See, e.g., the following reviews: A. Dalzell, *Phoenix* 17 (1963) 314-316; R. D. Williams, *CP* 58 (1963) 248-251; M. L. Clarke, *CR* 14 (1964) 43-45; P. T. Eden, *PVS* 3 (1963-64) 48-49; P. F. Hovingh, *Mnemosyne* 17 (1964) 425-426; and P. L. Smith, *Vergilius* 9 (1964) 15-19 in his review of E. L. Brown, *Numeri Vergiliani*.

and outline the general points presented against the mathematical thesis by reviewers.

Most Virgilians have become familiar with the Golden Section, the proportion achieved when a line is divided so that the smaller section is to the larger section as the larger section is to the line as a whole:  $m/M = M/(M + m) = \frac{1}{2}(\sqrt{5} - 1) = .618$ . The ratio is most readily approximated by the Fibonacci series, the discovery of Leonardo Pisano (*ca.* 1170–1230). The series consists of a sequence in which each number is the sum of the two numbers preceding, whatever the initial two numbers may be: e.g., 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, etc. The Golden Mean ratio, .618, will be produced by 21/34 and the numbers which follow in the series, so that if a passage of 21 lines is preceded or followed by a passage of 34 lines, they produce a Golden Mean ratio. Professor Duckworth is forced to assume not only that the Euclidean theorems were familiar to Virgil, but also that he knew their arithmetical equivalent and the Fibonacci series. At this point the reviewers begin to demur. A composite list of their comments on the present assumptions and other aspects of Professor Duckworth's thesis, supplemented by the comments of the present writer, would be as follows.

(a) There is no independent evidence that the Fibonacci series, so important for achieving the ratio, was known to Greek or Roman mathematicians. Leonardo Pisano appears to have been interested in Arabic mathematics. His *Liber Abaci* (1202), for example, helped to introduce Arabic numerals into Europe. If a source is to be sought for his series, it seems more likely that he derived it from his study of Arabic mathematics, than that he was influenced by Greek or Roman mathematicians or inherited knowledge current in Augustan Rome. In addition, Donatus' statement that Virgil studied mathematics in his youth is not proof of anything.

(b) There is no independent evidence that the Golden Mean ratio was part of contemporary literary aesthetic theory. It is mentioned by no ancient rhetorician or commentator. Although *De Divina Proportione* was the title of a work by Fra Luca Pacioli, published in 1509, the epithet "golden" is itself nineteenth century (e.g., Zeising, *Der goldene Schnitt*, 1884). The most obvious application of the ratio is in the visual arts and architecture, where it can be used to explain the beauty of certain proportions. Some scholars have in fact seen the Golden Section in classical temples and vases. There is, however, no evidence that a Roman listener would have recognized Virgil's use of the ratio.

(c) The extent to which Professor Duckworth finds the same ratio used by other Latin writers tells against the significance he assigns to it as part of Virgil's deliberate plan. None of the other poets concerned,

such as Lucretius, Catullus, and Horace, was noted as a student of mathematics. In addition, ratios have been found in non-rhyming English poetry from the sixteenth and twentieth centuries, even though there is no evidence that the poets concerned used the ratio deliberately.

(d) The argument that Ennius used the Golden Mean ratio, with proportions determined by the Fibonacci series, and that other Latin poets may have derived their practice from him, is highly unconvincing. Finding approximate ratios in fragments is unconvincing in itself and says nothing about the nature of the work as a whole.

(e) The method whereby the ratio is derived has been questioned. Since  $m/M = M/(M + m)$ , ideally one should be able to use either formula, translated into numerical terms. Thus, if the passages are 21 and 34 lines respectively,  $m/M = 21/34 = .618$ , and  $M/(M + m) = 34/55 = .618$ . If, however, the number of lines in each section does not correspond exactly with numbers in the Fibonacci series, the divergence from .618 varies according to the formula used. If, for example,  $m = 4$ , and  $M = 7$ ,  $m/M = 4/7 = .571$ , and  $M/(M + m) = 7/11 = .636$ . Professor Duckworth states that he allows ratios from .600 to .636, calculated by  $M/(M + m)$ , as approximate Golden Mean ratios, and (48) that it is unnecessary to list the ratio calculated by the simple formula,  $m/M$ . The implication seems to be that both formulae will produce ratios within the stated range, but this is not the case. Hence a question of methodology arises. Does one use the more complex formula,  $M/(M + m)$ , which is less sensitive to variation from .618, thereby increasing the number of ratios found to support the hypothesis, or does one use the simple formula,  $m/M$ , which will be a more rigorous test of validity? To anticipate slightly, it is perhaps worth noting that if the simple formula had been used to calculate the suggested proportions in Book One of the *Aeneid*, the total number within the .600 to .636 range would be 45. The complex formula produces 80.

(f) The significance of the ratios has been queried in several ways. M. L. Clarke (*CR* 14 [1964] 44) points out that because of the deviation of .018 allowed either way from .618, it is difficult to assess the importance of ratios in the larger sections. As he says, "we are not greatly impressed by the discovery of a Golden ratio between Books xi-xii (1862.2 lines) and Books vii-x (3255.1 lines) when we realize that one could equally well have been found if there had been some 300 more lines in xi-xii; nor by a similar discovery in respect of Books ix-xii (3575.3 lines) to Books i-viii (6276.9 lines) when the Golden ratio would also appear if the total number of lines in the last four books was greater by 600 lines." It has also been noted that the majority of the ratios appear in short passages where approximation to .618 is relatively easy to find. It is

easier, for example, to find 3 lines juxtaposed with 5 than to find units of 89 and 144. The test of validity must lie in the longer passages, where in fact suggested approximate ratios are less frequent. If Virgil had deliberately used the Golden Proportion, one would expect to find more examples of exactly .618 than 45 out of 1044, and also that more than 3 would appear in the main divisions of the work.

(g) It is difficult to determine the likelihood of chance occurrence of the Golden Proportion. Virgil obviously had a strong and sensitive feeling for asymmetry in the sections of his narrative. The probability of fortuitous ratios would seem, therefore, to be fairly great.

(h) There is dispute about the validity and significance of some of the paragraph or subject units used. In some cases divisions appear arbitrary, if not actually chosen to suit the theory.

No one would dispute that the relationship between sections of Virgil's narrative can be described mathematically, or that divisions of the narrative made by commentators are to a large extent subjective. At times, however, Professor Duckworth seems to exalt mathematical form to the extent that it becomes divorced from content. Concerning Table IV, for example, 4 or 5 part episodes, he comments "What is most surprising about the ratios . . . is the fact that the parts of the major and/or minor are so often linked not only mathematically but by the content as well, by a similarity of theme or idea, by the identity of speakers, or by the alternation of speech and narrative" (54). For mathematical descriptions to be convincing or useful, they must emerge from such relationships of content as Professor Duckworth describes. Naturally structure was an important consideration for Virgil, but it was neither his only preoccupation nor his chief concern. R. D. Williams' comments perhaps restore perspective:

an epic poet is a writer first, and he builds his poem with words which have meaning, as bricks and stones do not. He must be concerned with many things more immediately intrinsic to his craft than structure—with the values and meaning of the poem, with the development of the story, with the use of imagery and symbol, with the sound of his music, with emotion and reason, with passion and suffering. For Virgil, his love of Rome, his sorrow at suffering, his many cadenced hexameter, his Dido, his Turnus, his Aeneas were urgent matters of which structure was the servant, not the master.<sup>5</sup>

Unless it can be shown that the mathematical ratios serve these wider interests, whatever their fascination may be they are of little concern to the serious student of Virgil.

With these general points in mind, we can turn to the narrative of *Aeneid* 1. Professor Duckworth divides the book as a whole into three main sections, each having two or three sub-sections, as follows (176):

<sup>5</sup>See review cited above, n. 4, 248.

- I. 1a-222, Prologue. Juno and the Storm
  - 1a- 80, Prologue. Juno and Aeolus
  - 81-156, The Storm and Neptune
  - 157-222, Aeneas and the Trojans in Africa
- II. 223-417, The Venus Episodes
  - 223-296, Venus and Jupiter
  - 297-417, Venus and Aeneas
- III. 418-756, The Trojans at Carthage
  - 418-519, Carthage and Dido
  - 520-656, The Trojans welcomed
  - 657-747, Cupid and the Banquet
  - 748-756, Epilogue to introduce *Aeneid* 2-3.

Some aspects of his arrangement might be queried. The basis for including the prologue in a subject unit with the conference between Juno and Aeolus is not clear. While verses 1-33 are relevant to the specific instance of Juno's hostility which follows, their implications are not limited simply to the meeting of the divinities: rather, there is a much wider statement of theme for the poem as a whole: *tantae molis erat Romanam condere gentem*.

Within the prologue itself, the inclusion of verses 1a-d is problematical. Although some commentators have judged the verses to be Virgilian on stylistic and metrical grounds, there are equally good arguments from content and presentation for considering them an interpolation. In no other reference to his poetry does Virgil give simply a list of works. The ending of *Georgics* 4, cited as supporting evidence (84), is not comparable since it deliberately contrasts the poet with the man of action, Caesar, and, by reference to the first *Eclogue*, perhaps suggests the vulnerability of those who must try to maintain the arts of peace *Caesar dum magnus ad altum/fulminat Euphraten bello*.<sup>6</sup> Secondly, such extended personal comment is out of character with the unobtrusiveness of the poet throughout the epic. The personal references *cano* (1e) and *mihi causas memora* (8) are, however, unobtrusive, little more than functional words with an appropriate Homeric aura. The assertion that the lines are proved Virgilian by the fact that more exact ratios are produced by their inclusion is not strong evidence unless one agrees that the .618 thesis has been proved.

<sup>6</sup>*Haec super arborum cultu pecorumque canebam  
et super arboribus, Caesar dum magnus ad altum  
fulminat Euphraten bello victorque volentis  
per populos dat iura viamque adfectat Olympo.  
illo Vergilium me tempore dulcis alebat  
Parthenope studiis florentem ignobilis oti,  
carmina qui lusi pastorum audaxque iuventa,  
Tityre, te patulae cecini sub tegmine fagi.*

Finally, it is not entirely clear why Juno's soliloquy and her meeting with Aeolus should be separated from 81–156, the storm and Neptune. Juno's anger leads to the storm and Aeolus is her willing agent in rousing it. Virgil has joined the two passages by the transitional verses 81–82, *Haec ubi dicta, cavum conversa cuspide montem/impulit in latus*. Possibly a tripartite division consisting of 1e–33, Prologue, 34–156, The Storm, and 157–222, Aeneas and the Trojans in Africa, would have been more appropriate.

In the second major grouping, the Venus episodes, the inclusion of verses 297–304, Jupiter's sending of Mercury to influence Dido, with the meeting of Venus and Aeneas seems somewhat forced. While it might be argued that both scenes show the interaction of human and divine beings, and that the creation of a favourable frame of mind in Dido is directly relevant to Venus' sympathetic account to Aeneas of Dido's previous trials, there are also good reasons for considering the Mercury episode part of the scene between Venus and Jupiter, and a transitional passage from one level of narrative to another. Jupiter had been gazing down at Libya when Venus approached him. His answer to her questions includes both prophecy and reassuring practical action, the dispatching of Mercury to Dido, which again returns the scene to Libya. The Mercury scene begins with a reference to Jupiter (297) *Haec ait et*, and maintains the supra-human perspective of the meeting between Venus and Jupiter (302, *et iam iussa facit*). In 305, *At pius Aeneas per noctem plurima volvens*, we return to Aeneas whom we left at 220–222 wondering about the fate of his companions. The introductory word *at*, I suggest, has the general effect "and meanwhile, while all this was taking place," and does not merely connect the Venus and Aeneas scene with the preceding passage, 297–304.

Finally, Professor Duckworth feels that the third division of the book ends at 747, as the company applauds the song of Iopas, and that the last nine lines are the introduction to *Aeneid* 2–3. Several points could be raised here. If the epilogue is not considered part of the third division, why is the prologue treated as part of the first main division? The presence of Cupid, who has been active during the feast, is still felt during the lines in question (749, *Dido longum . . . bibebat amorem*) thus linking them closely with the preceding narrative. Dido's request for Aeneas' story is introduced by the phrase *nec non et* (748), which seems a strong linking phrase. Thus it is perhaps an over-simplification of Virgil's narrative technique to ignore the double function which the ending of the book serves. Conversation is as integral a part of the feast as toasts and the song of Iopas, and as important for Dido's characterization as earlier narrative comment. The feast, in effect, overflows into *Aen.* 2.1–13, Aeneas' reply to Dido.

Within the larger narrative units outlined, it is sometimes difficult to agree with the suggested subdivisions of subject matter. The methods of division used are relatively straightforward. Ratios are found within single speeches, between two or more speeches, between speech and narrative passages, between sections of narrative or description, between non-contiguous passages consisting of speeches or episodes which frame a central passage, or between interlocking passages which have some relation in content. As applied to Book One, however, the method can be faulted on the general ground of inconsistency. "Speech" sometimes means direct speech only, at other times speech plus its narrative introduction. A passage may be considered as part now of one scene, now of another to arrive at different sets of ratios. In some cases only a section of a scene or speech is used so that the significance of the ratio attained seems questionable in terms of content. In others it is not immediately obvious why passages are combined or juxtaposed. Detailed consideration of the ratios suggested for 1. 1a-222, "Prologue. Juno and the Storm," will provide the evidence for these comments, and selected examples from other sections of the book will indicate specific difficulties in Professor Duckworth's scheme.

1. 1a-80 PROLOGUE. JUNO AND AEOLUS (ratios 1, 353, 470, 711, 712; ratios including other sections 2, 354, 471, 942)<sup>7</sup>

(i) *Treatment of The Prologue*

Two general questions have been asked: the validity of linking the

<sup>7</sup>References for the ratios will be given in the following way: 1. number in Duckworth's Tables; 2. line references for the major; 3. line references for the minor; 4. the ratio calculated by the simple formula; 5. the ratio calculated by the complex formula.

Number	M	m	m/M	M/(M + m)
1	34- 75	8- 33	.619	.618
353	50- 64	34- 49	.615	.619
	65- 75			
470	8- 22	23- 32	.625	.615
	33			
711	1a- 7	8- 11	.606	.623
	12- 33	34- 49		
712	12- 33	1a- 11	.585	.631
	50- 80	34- 49		
2	34- 75	76-101	.619	.618
354	76- 80	92-101	.625	.615
	81- 91			
471	34- 75	76-123	.640	.610
	124-156			
942	81-156	1a- 80	.592	.628
	157-222			

prologue specifically with the conference of Juno and Aeolus, and of including 1a-d as part of the prologue. If the disputed verses are omitted, ratios 711, 712, and 942 would be adversely affected.

Ratio 470 depends upon the nature of the relationship within verses 8-33. In 8-11, *Musa mihi causas memora . . .*, the poet asks the reason for the ferocity of Juno's anger against the Trojans, and in 12-33, *Urbs antiqua fuit . . . tantae molis erat Romanam condere gentem*, the answer is given in terms of her devotion to a Carthage threatened with future destruction by Aeneas' descendants, and her lingering resentment at previous insults by the Trojans. If the preceding comments are valid, there are insufficient grounds for considering 8-22 as a unit; for making a break between 22 and 23, (*populum*) *venturum excidio Libyae: sic volvere Parcas./id metuens*; and least of all for separating 33 from the passage for which it forms an impressive and fitting climax. Thus ratio 470 is invalid, and alternative groupings will not produce approximate Golden Mean ratios.

Ratios 711 and 712 combine 8-11 and 1a-11 with Juno's soliloquy, 34-49, to form the minor, although the soliloquy and the meeting with Aeolus are more obviously part of the "storm" scene, and the perspective has shifted from general statements to the immediacies of narrative. This arrangement fails to carry conviction.

(ii) *Where does the conference between Juno and Aeolus end?*

Professor Duckworth begins the meeting of Juno and Aeolus at 34 and ends it with the conclusion of Juno's speech at 75 (except for ratio 942 and the general divisions of narrative given above, where the scene ends at 80). As a conference between divinities, the scene would be singularly ineffective without Aeolus' affirmative reply (76-80). That the lines are an integral part of the scene is suggested also by the introduction of 76, *Aeolus haec contra*. The transition to the account of the storm begins in 81-82, when Aeolus strikes the rock, a shift in perspective from divine conference to its effect upon the natural world and human beings. The inclusion of Aeolus' speech in the Juno and Aeolus scene would produce the following alternative ratios:

	$m/M$	$M/(M + m)$
1.	.553	.644
2.	.447	.691
353.	.516	.660
354.	.909	.524
471.	.538	.650



2. 81-156 THE STORM AND NEPTUNE (ratios 4, 5, 6, 7, 8, 472, 474, S997; ratios including other sections 3, 473)<sup>8</sup>

Virgil's account of the storm is a carefully wrought narrative with effective use of stylistic devices and skilful transitions in perspective. Since the validity of 10 ratios depends upon the manner in which the scene is analysed, it should be considered in some detail.

In verses 81-87, the first line and a half introduced by *Haec ubi dicta* acts as a transition from the conference. The winds sweep the land (82-83) and settle upon the sea, rolling waves against the shore (84-86). Their effect upon ships and men is given in 87 in general terms (*insequitur clamorque virum stridorque rudentum*) which, while part of the panorama of the storm, makes a transition from natural description to its effect upon humans. The perspective becomes specifically that of the Trojans rather than the narrator at 88, and from 88 to 91, the storm is presented in terms of what they see and feel—clouds, thick darkness, thunder, lightning, fear. In 92-101 the perspective narrows further to Aeneas' despair and his wish that he had died at Troy. The effect of the storm on the ships is described in 102-123, a passage which appears to have as its basic structural principle the alternation of specific and general examples. First Aeneas' ship is struck (102-105). The following passage (106-112) is general (*hi . . . his: tris . . . tris*). The destruction of Orontes' ship which Aeneas sees *ipsius ante oculos* is described in 113-119, and in 120-123 the accumulation of individual examples (*iam . . . Ilionei,/ iam . . . Achatae*) blends into the comprehensive *omnes accipiunt . . . imbrem*. It would appear, then, that there are three parts to 81-123, the general description of the storm, the Trojans' and Aeneas' reactions to it, and its effect upon the ships. Neptune's intervention is also presented in three closely related sections: Neptune sees the storm (124-130);

<sup>8</sup> Number	<i>M</i>	<i>m</i>	<i>m/M</i>	<i>M/(M + m)</i>
4	106-112	102-105	.571	.636
5	124-156	102-123	.667	.600
6	113-119	120-123	.571	.636
7	131-141	124-130	.636	.611
8	148-156	142-147	.667	.600
472	81- 83 92-101	84- 91	.615	.619
474	106-119	102-105 120-123	.571	.636
S997	81-101 113-123 142-156	102-112 124-141	.618	.618
3	76- 80	81- 83	.600	.625
473	81-101 157-222	102-156	.632	.613

he rebukes the winds (131-141); and he calms the storm and rescues the ships (142-156).

In terms of the preceding outline, the following comments might be made on the ratios listed. For ratio 3, verses 81-83 seem an invalid grouping since they do not include 84-87 which also describe the storm in general terms. Alternative ratios including these verses would be .833 and .546, both unacceptable. Ratio 472 could be considered invalid on the same grounds. Ratios 4 and 6 use only parts of the description of the plight of the ships, thereby negating what seems to be a deliberate pattern of alternating examples, while ratio 474 is based on a less convincing tripartite pattern. Ratio S997 separates 102-112 and 113-123, both of which describe the effect of the storm on the ships, and links the former with Neptune's becoming aware of the storm and rebuking the winds (124-141), and the latter with the general description and Trojan reaction (81-101), and Neptune's calming the storm (142-156). The reasons for this arrangement are not immediately obvious. In contrast, ratio 473 links 102-123, the "ship" passage, with Neptune's intervention, but separates the lines from 81-101 to which they are more closely connected verbally (102 begins *talìa iactanti*, referring to Aeneas) and in terms of perspective. In Ratio 8, the division of 142-156 into two sections seems unwarranted since it separates the simile of the statesman from its context in the preceding lines. Ratios 5 and 7 are perhaps of doubtful significance since they comprise only sections of scenes: ratio 5, verses 102-156, omits the initial section from verse 81, and ratio 7, verses 124-141, omits 142-156, Neptune's calming of the storm. Finally, if the simple formula had been used, ratios 4, 5, 6, 8, and 474 would have been outside the .600-.636 range.

### 3. 157-222. AENEAS AND THE TROJANS IN AFRICA (ratios 9, 10, 11, 12, 13, 355, 713, S998, S999)<sup>a</sup>

The ratios for the present section depend generally upon two units of

<sup>a</sup> Number	<i>M</i>	<i>m</i>	<i>m/M</i>	<i>M/(M + m)</i>
9	162-169	157-161	.625	.615
10	174-179	170-173	.667	.600
11	187-197	180-186	.636	.611
12	198-203	204-207	.667	.600
13	214-222	208-213	.667	.600
355	157-197	198-207 208-222	.609	.621
713	157-169 180-197	170-179 198-207	.645	.608
S998	162-169 174-179	157-161 170-173	.643	.609
S999	187-197 198-203	180-186 204-207	.647	.607

narrative, the landing in Africa and initial preparations for a meal (157–179), and Aeneas' unsuccessful search for his companions, his killing of the stag, and the feast (180–222).

For ratios 9 and 10, 157–179 are considered as four sections. The division of 157–169 with a new section beginning at 162 (*hinc atque hinc vastae rupes geminique minantur/in caelum scopuli*) would seem to destroy the logic with which the harbour is presented, beginning at 159, *est in secessu longo locus*. There is a steady progression of detail from the island which breaks the force of the waves, to the high cliffs above calm water, trees, caves, fresh spring water, and quiet anchorage, so that, in essence, there is one section of 13 lines. Similarly, it could be argued that a new section does not begin at 170 (*huc septem Aeneas . . . / . . . subit*) since the use of the demonstrative *huc* links the actual landing closely with the preceding sentence (168), *hic fessas non vincula navis/ulla tenent*. Thus the transition from natural description to human activity within the setting is accomplished without a break in the chain of narrative.

Ratios 11, 12, and 13, verses 180–222, are based on simple division into speech or narrative, and further subdivision of the resulting sections. This method seems to produce some distortions of the text: 197. . . . *et dictis maerentia pectora mulcet*, which ends the narrative of ratio 11, is closely related to Aeneas' speech of ratio 12 which follows it, and 208–209, *Talia voce refert curisque ingentibus aeger/spem vultu simulat, premit altum corde dolorem*, which begins the narrative for ratio 13, the feast, cannot be other than highly relevant to Aeneas' preceding words of comfort to his followers. Aeneas is a leader who cares not only for the physical needs of his followers but also for their spiritual needs, and his methods meet with some success. If the passage is to be divided into sections, surely these points must be considered. Thus the shooting of the stag, the return to camp and Aeneas' speech would provide one unit (180–209), and the feast and preparations for it would form another, 210, *illi se praedae accingunt*, to 222 which ends the scene. Aeneas is the focal point of the scene until 210, and any division of the text which obscures his central role seems lacking in force.

There are, then, logical alternatives to Professor Duckworth's subject divisions which do not produce Golden Mean ratios, and in some cases his groupings must be considered invalid. Problems presented by the suggested ratios for the remaining sections of Book One can be dealt with more briefly.

First, there are 19 ratios, in addition to those already mentioned, which might have been excluded since they fall within the .600–.636 range only if they are calculated by the complex formula (16, 25, 28, 356,

357, 360, 363, 476, 478, 479, 480, 482, 717, 718, 719, 723, 724, 981, S1000).<sup>10</sup> Secondly, there are incomplete groupings which use only parts of a speech or narrative passage, and therefore indicate little about the speech or the passage as a whole (e.g., ratios 21 and 22, which use only 522-543 of Ilioneus' speech, and exclude 544-558). Thirdly, some combinations of passages seem to bear little relationship to subject-matter (e.g., ratio 356, in which 297-304, Jupiter sends Mercury to Dido, and 305-313, Aeneas decides to explore, are major, and 314-324, the disguised Venus appears to Aeneas and addresses him, is minor). Conversely, logically related passages are frequently separated to form parts of major and minor respectively (e.g., 358 which separates the splendid simile of the bees (430-436), from the description of Carthage (418-429), and combines it with the description of the grove and temple

<sup>10</sup> Number	<i>M</i>	<i>m</i>	<i>m/M</i>	<i>M/(M + m)</i>
16	393-401	387-392	.667	.600
25	613-630	631-642	.644	.608
28	740b-747	736-740a	.579	.633
356	297-304 305-313	314-324	.647	.607
357	402-417 418-440	441-465	.641	.609
360	520-578 579-642	643-722	.661	.602
363	695-706 707-711	712-722	.647	.607
476	335-370a	326-334 372-385a	.638	.610
478	370b-377 384-385a	378-383	.652	.605
479	430-440 450-452	441-449	.643	.609
480	466-473 485-493	474-484	.647	.607
482	740b-752	736-740a 753-756	.667	.600
717	326-334 335-370a	372-385a 387-401	.644	.608
718	418-429 441-449	430-440 450-452	.667	.600
719	418-440 453-493	441-452 494-519	.594	.627
723	546-550 555-558	544-545 551-554	.667	.600
724	561-571 582-585	572-578 579-581	.667	.600
981	418-747	223-417	.594	.627
S1000	305-313 325-334	314-324	.579	.633

of Juno (441-449). Brief narrative passages are omitted although they are an integral part of a scene predominantly consisting of direct speech. For ratio 14, e.g., 227-297, *illum*, 227, makes no sense without its antecedent *Iuppiter* in 223; and in 326-385*a*, ratio 476, which again consists of direct speech, it is necessary to ignore 370*b*-371, one and a half lines of narrative within the passage, in order to produce a valid ratio.

Of the 80 ratios listed for Book One, 8 appear to satisfy the criterion of mathematical structure reinforcing content: ratios 7, 20, 359, 361(?), 362, 727, 943, and S1001.<sup>11</sup> In all cases passages are combined or contrasted so as to reflect sensitively or to enhance the interplay of theme, personality, description, or image.

A number of conclusions can be drawn from this study of *Aeneid* 1. It is clear that no consistent pattern of Golden Mean ratios so far suggested for Book One can be generally convincing. Professor Duckworth fails in his attempt because his mathematical analysis does not usually correspond to or support the artistic principles on which the first book is constructed. Since there does not appear to be a consistent pattern of .618 ratios in Book One, it seems unlikely that consistent patterns exist in the other books of the *Aeneid*. Certainly Virgil has not declared his mathematical interests by using perfect ratios at the beginning of his work. The general criticisms discussed earlier are borne out by detailed examination.

Perhaps the chief failing in Professor Duckworth's thesis is that he is attempting to put Virgil into a strait jacket, and to find something which is not characteristic of the author. The phrase "transitional passage" has been used of the ending of *Aeneid* 1 and the Mercury scene. It is more typical of Virgil that one scene blends into the next than that narrative passages are rigidly delimited. One might add that in the *Aeneid* no issues are clearly defined. Destruction in curious fashion leads to rebirth. Every positive action has negative undertones, and each success seems to be accompanied by human loss and suffering. To

<sup>11</sup> Number	<i>M</i>	<i>m</i>	<i>m/M</i>	<i>M/(M + m)</i>
7	131-141	124-130	.636	.611
20	453-493	494-519	.634	.612
359	520-560	561-578	.633	.612
		579-585		
361	613-630	586-612	.619	.618
	631-656			
362	695-722	657-694	.613	.620
	723-756			
727	657-694	643-656	.633	.613
	712-722	695-711		
943	297-417	223-296	.612	.621
S1001	520-560	561-585	.638	.615
	613-656	586-612		

present his expression of human and national experience, Virgil developed an evocative language which depends very greatly upon the interplay of associations. Archaisms, literary allusions, words and phrases which gather connotations as they reappear, all expand the scope of his poetry by taking it beyond the limits of individual lines and scenes. To cope with such a fluid medium, Virgil required a controlled structure, not a rigid one.

Here we might return to a point made earlier concerning Virgil's feeling for asymmetry. One does not expect a writer with any feeling for language to compose a long poem in units of equal length, or, within those units, to produce phrases and sentences of approximately the same length and structure. The effect would be monotonous. Great fluctuations, too, in the length of units and subsections ultimately become bewildering. The effect would be disjointed, and intense, and would tend to contrast rather than to draw passages together. Such a style, although it might be used by a political speaker in a moment of strong emotion, is inappropriate to the subject-matter of the *Aeneid*. There remains the style between these two extremes, the imbalance which produces tension between units and relates them. A passage of two groups of 5 lines, for example, is somewhat stolid, but the combination of 6 and 4 is structurally interesting. Professor Duckworth has seen that Virgil is sensitive to structure, and that asymmetry is one of the principles upon which the work is constructed. The error consists in thinking that Virgil's varied and skilfully interwoven narrative patterns can be reduced to a formula.

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